EDEXCEL

190 High Holborn London WC1V 7BH

January 2005

Advanced Subsidiary/Advanced Level

General Certificate of Education

Subject: **Pure Mathematics** **Paper: P3**

Question Number	Scheme		Marks	
1. (a)	$f(-2) = -16a - 4a + 6 + 7$	Attempt $f(\pm 2)$	M1	
	$f(-2) = -3 \Rightarrow -20a + 13 = -3$	Solve eqn. $f(\pm 2) = -3 \to Ka = L$	M1	
	i.e. $20a = 16$			
	$\underline{a = \frac{4}{5}}$	o.e.	A1	(3)
(b)	$f(\frac{1}{2}) = \frac{a}{4} - \frac{a}{4} - \frac{3}{2} + 7 = \underline{\frac{11}{2}}$ (o.e.)	Attempt $f(\pm\frac{1}{2})$	M1	
		$\frac{11}{2}$ or exact equiv.	A1	(2) 5
2. (a)	$\sin(3x + x) = \sin 3x \cos x + \cos 3x \sin x$	Use of a correct formula $\ldots\pm\ldots=\ldots\pm\ldots$	M1	
	$\sin(3x - x) = \sin 3x \cos x - \cos 3x \sin x$	Both correct	A1	
	(Subtract) $\Rightarrow$ $\underline{\sin 4x - \sin 2x = 2\sin x \cos 3x}$	$p = 4, q = 2$	A1 cso.	(3)
(b)	$\int 2\sin x \cos 3x \, dx = \int (\sin 4x - \sin 2x)\,dx$	Attempt $\int$ using a $\sin px \to \pm \frac{\cos px}{p}$	M1	
	$= \underline{-\frac{\cos 4x}{4} + \frac{\cos 2x}{2} + c}$	$\int$ their p, q	A1 $\int$	(2)
(c)	$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} 2\sin x \cos 3x \, dx = \left(-\frac{1}{4}\cos\frac{10\pi}{3} + \frac{1}{2}\cos\frac{5\pi}{3}\right) - \left(-\frac{1}{4}\cos 2\pi + \frac{1}{2}\cos\pi\right)$		M1	
	$= \underline{\frac{9}{8}}$		A1	(2) 7
3. (a)	$x^2 + y^2 - 12x + 4y + 20 = 0$	Attempt to complete square	M1	
	$(x-6)^2 + (y+2)^2 + k = 0$	Centre $\underline{(6, -2)}$	A1	(2)
(b)	$(x-6)^2 + (y+2)^2 = 20$	$k = -36 - 4 + 20$ o.e.	M1	
		Radius $= \underline{\sqrt{20}}$	A1	(2)
Use of Formulae (a)	$2g = -12, \; 2f = 4, \; c = 20$ Centre $(-g, -f)$	$(\pm 6, \pm 2)$	M1	
		Centre $\underline{(6, -2)}$	A1	(2)
(b)	Radius $= \sqrt{36 + 4 - 20}$	36, 4, 20 and $\sqrt{\ }$	M1	
		Radius $\underline{\sqrt{20}}$	A1	(2)
(c)		P.T.O.		

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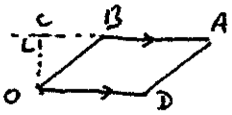
3. (c)	$x^2 - 12x + 20 = 0$ Sub $y=0$ in $C_1 \to$ 3TQ $=0$	M1
	$\Rightarrow x = 2, 10$	A1
	Centre of C_2 is $\underline{(6,0)}$	B1
	Radius of C_2 is $6-2$ or $10-6$ $\underline{=4}$ ∫ their centre, A or B	B1∫
	Equation of C_2 is $\underline{(x-6)^2 + y^2 = 4^2}$ o.e.	M1 ∫ Centre and radius
		A1 (6) 10
4. (a)	$(1-x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(-x)^3$	M1 A1
	$\left(= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 \dots\right)$	
	$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}x^3$ Ms need attempt at binomial → at least x^2 term.	M1 A1
	$\left(= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 \dots\right)$	
	Subtract	M1
	$\therefore f(x) = \underline{\frac{1}{2}x^2 + \frac{1}{4}x^3}$ c.a.o.	A1 (6)
(b)	$f'(x) = x + \frac{3}{4}x^2 \dots$, $f''(x) = 1 + \frac{3}{2}x \dots$ Attempt f' & f''	M1
	$f(0) = 0$ and $f'(0) = 0$	B1
	$f''(0) > 0$, $\therefore$ Minimum at origin ✱	M1, A1 cso. (4) 10
5. (a)	$\vec{AB} = \underline{b} - \underline{a} = \begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix}$; $\therefore$ Equation of L $\underline{\underline{r} = \begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix}}$ (o.e.)	M1 ; A1 (2)
(b)	$\begin{pmatrix} 3t \\ 5-3t \\ 5-6t \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix} = 0$ Attempt $\vec{OC}$ in terms of t	M1
	$\vec{OC} \cdot \vec{AB}$ attempt	M1
	$\Rightarrow 9t - 15 + 9t - 30 + 36t = 0$ Evaluation of $\cdot$ product	M1
	i.e. $t = \frac{5}{6}$ $t =$	A1
	$\therefore \underline{\vec{OC} = \begin{pmatrix} 2.5 \\ 2.5 \\ 0 \end{pmatrix}}$ $\vec{OC} =$	A1 (5)
(c)	 $\vec{OD} = \vec{BA} = \underline{a} - \underline{b}$ or $-\vec{AB}$, $= \underline{\begin{pmatrix} -3 \\ 3 \\ 6 \end{pmatrix}}$	M1, A1 (2)
(d)	$\lvert\vec{OC}\rvert = 2.5\sqrt{2}$; $\lvert\vec{OD}\rvert = 3\sqrt{1^2+1^2+2^2} = 3\sqrt{6}$ Attempt at least one relevant length	M1
	both correct	A1
	Area $= \lvert\vec{OC}\rvert \times \lvert\vec{OD}\rvert$ (o.e.) , $= 7.5\sqrt{12}$ or $\underline{15\sqrt{3}}$ or AWRT 26.0	M1, A1 (4) 13

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6. (a)	$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{3}{2t}$	Attempt $\frac{dy}{dx}$	M1	
	Gradient of normal is $-\frac{2t}{3}$	Use of $\perp^r$ rule	M1	
	At P $t = 2$	$t = 2$ @ P	B1	
	∴ Gradient of normal @ P is $-\frac{4}{3}$		A1	
	Equation of normal @ P is $y - 9 = -\frac{4}{3}(x - 5)$		M1	
	Q is where $y = 0$ ∴ $x = \frac{27}{4} + 5 = \underline{\frac{47}{4}}$ (o.e.)		A1	(6)
(b)	Curved Area $= \int y\,dx = \int y \frac{dx}{dt}\,dt$	Attempt change of var.	M1	
	$= \int 3(1+t).2t\,dt$		A1	
		$\int x^n \to x^{n+1}$	M1	
	$= [3t^2 + 2t^3]$		A1	
	Curve cuts x-axis when $t = -1$	$t = -1$	B1	
	Curved Area $= [3t^2 + 2t^3]_{-1}^{2} = (12 + 16) - (3 - 2)$ $(= 27)$	Use of their $t = -1, t = 2$	M1	
	Area of $\triangle PQ$ triangle $= \frac{1}{2}((a) - 5) \times 9$ $(= 30.375)$		M1	
	Total area of R = Curved Area + Δ		M1	
	$=$ **57.375** or AWRT $\underline{57.4}$		A1	(9)
				(15)

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7. (a)	$I = \int x \operatorname{cosec}^2\left(x+\frac{\pi}{6}\right)dx = \int x \, d\left(-\cot\left(x+\frac{\pi}{6}\right)\right)$	Attempt Parts	M1	
	$= -x\cot\left(x+\frac{\pi}{6}\right) + \int \cot\left(x+\frac{\pi}{6}\right)dx$	.... $+\int \cot\left(x+\frac{\pi}{6}\right)dx$	A1	
	$= \underline{-x\cot\left(x+\frac{\pi}{6}\right) + \ln\left\lvert\sin\left(x+\frac{\pi}{6}\right)\right\rvert + c}$ ⊛		A1 c.s.o.	(3)
(b)	$\int \frac{1}{y(1+y)}\,dy = \int 2x \operatorname{cosec}^2\left(x+\frac{\pi}{6}\right)dx$	Attempt to separate	M1	
	LHS $= \int\left(\frac{1}{y} - \frac{1}{1+y}\right)dy$	Attempt P.F. on LHS	M1	
		Correct P.F.	A1	
	$\therefore \ln y - \ln\lvert 1+y\rvert$ or $\ln\left\lvert\frac{y}{1+y}\right\rvert = 2$ (a)	LHS some correct $\ln\int$	M1	
		Use of (a) in RHS	M1	
	$\therefore \underline{\frac{1}{2}\ln\left\lvert\frac{y}{1+y}\right\rvert = -x\cot\left(x+\frac{\pi}{6}\right) + \ln\left\lvert\sin\left(x+\frac{\pi}{6}\right)\right\rvert + c}$ ⊛		A1 c.s.o.	(6)
(c)	$y=1, x=0 \Rightarrow \frac{1}{2}\ln\frac{1}{2} = \ln\left(\sin\frac{\pi}{6}\right) + c$	Use of $y=1, x=0$	M1	
	$\therefore c = -\frac{1}{2}\ln\frac{1}{2}$		A1	
		Sub $x = \frac{\pi}{12}$	M1	
	$x = \frac{\pi}{12} \Rightarrow \frac{1}{2}\ln\left\lvert\frac{y}{1+y}\right\rvert = -\frac{\pi}{12}\cdot 1 + \ln\frac{1}{\sqrt{2}} - \frac{1}{2}\ln\frac{1}{2}$	$\sqrt{\text{"}c\text{"}}$	A1	
	(i.e. $\ln\left\lvert\frac{y}{1+y}\right\rvert = -\frac{\pi}{6}$)			
	$\frac{y+1}{y} = e^{\frac{\pi}{6}}$ (o.e.)	Out of ln	M1	
	$1 = y\left(e^{\frac{\pi}{6}} - 1\right)$			
	$\therefore y = \underline{\frac{1}{e^{\frac{\pi}{6}} - 1}}$ (o.e.)	A correct $y=$ expression	A1	(6)
				(15)

G.B. Attwood. 29/10/04.

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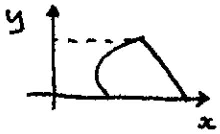
	ALTERNATIVE SOLUTIONS	
4 (a)	$f(x) = \frac{1-\sqrt{1-x^2}}{\sqrt{1-x}}$	M1
	$(1-x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \dots$	A1
	$1-(1-x^2)^{\frac{1}{2}} = 1 - \left(1-\frac{1}{2}x^2\dots\right) = \frac{x^2}{2}\dots$ Use of binomial to attempt both	M1 A1
	$f(x) = \frac{x^2}{2}\left\{1+\frac{x}{2}+\dots\right\}$ Multiply	M1
	$= \frac{x^2}{2} + \frac{x^3}{4}$	A1 (6)
6. (a).	Cartesian Equation: $(y-3)^2 = 9(x-1)$ or $y = 3+3\sqrt{x-1}$	B1
	$\frac{dy}{dx} = \frac{3}{2\sqrt{x-1}}$	M1
	[Rest as in scheme]	
(b)	$x = 2\ (y=0)$, $x = 1\ (y=3)$	B1
	(Two functions) $\pm$	M1
	Curved Area $= \int(3+3\sqrt{x-1})\,dx - \int(3-3\sqrt{x-1})\,dx$	A1
	[Rest as in scheme]	
ALT	 Trapezium $- \int x\,dy$ $= \frac{1}{2}9(5+11.75)$	M1
	Limits $^{9}_{0}$	B1
	$\int x\,dy = \frac{1}{9}\int[(y-3)^2+1]\,dy$ Attempt $x = x(y)$	M1
	correct integral	A1
	[Rest as in scheme + M1 for $\int + \Delta$ is for Trap $- \int$]	